For the case of a homogeneous soil, characterized by the critical velocity v_0, as follows from the results of [3], this limit as $h \to \infty$ is equal to $2q/v_0$ (the values of h, q, and v_0 are dimensional).

LITERATURE CITED

1. É. B. Polyak and E. N. Sher, "The form of the ejection funnel with the explosion of a fuse-type discharge in a two-layer medium," Zh. Prikl. Mekh. Tekh. Fiz., No. 2, 143-146 (1973).
2. V. M. Kuznetsov, "The form of the ejection funnel with an explosion at the surface of the ground," Zh. Prikl. Mekh. Tekh. Fiz., No. 3, 152-156 (1960).
3. V. M. Kuznetsov and É. B. Polyak, "Pulsed-hydrodynamic schemes with ejection by the explosion of fuse-type charges," Fiz.-Tekh. Probl. Razrab. Polezn. Iskop., No. 4, 32-39 (1973).
4. P. F. Fil'chakov, "Determination of the constants of the Christoffel–Schwarz integral using modeling on electrically conducting paper," Ukr. Mat. Zh., 13, No. 1, 72-79 (1961).

PROPAGATION OF A TWO-DIMENSIONAL PLASTIC WAVE IN A MEDIUM WITH NONLINEAR UNLOADING

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The plane stationary problem about the effect of a moving load on a nonlinearly compressed half-plane is considered.

The case of linear loading and unloading of the medium has been examined in [1, 2]. The solution of the problem obtained in [1] by conformal mapping holds in the case when the propagation velocity a_1 of the unloading strains exceeds the velocity of loading motion. This problem is solved in [2] without constraints on the velocity a_1 by the Mellin integral transform method for the triangular load case.

The influence of nonlinear properties of the half-plane material on the propagation of shock-wave processes therein is studied by the numerical method of characteristics and analytically in this paper.

The computational scheme proposed can be used to determine the parameters of an inhomogeneous medium for different profiles of a given load.

Let a decreasing normal load move monotonically at a constant velocity D exceeding the propagation velocity of the loading–unloading strains of the medium over the surface of a half-plane. The load profile does not change as the wave propagates. The medium filling the half-plane possesses such mechanical properties that the relation between the pressure p and the volume strain ε is nonlinear and irreversible during loading and unloading, where $dp/d\varepsilon > 0$, $d^2p/d\varepsilon^2 > 0$ and the slope of the unloading branch of the $p \sim \varepsilon$ diagram exceeds the slope of the loading branch.

In this case, a shock with the curvilinear surface Σ will be propagated in the half-plane, and the perturbation domain will be bounded by the front Σ and the boundaries of the half-plane. It is assumed that the medium is loaded instantaneously at the front Σ, while unloading occurs in the perturbed domain behind the front. The relationships

$$\rho_0 a = \rho^* (a - v_n^*), \quad \rho_0 a v_n^* = p^*, \quad v_\tau^* = 0 \qquad (a = D \sin \alpha) \tag{1}$$

hold on the surface of strong discontinuity Σ from the mass and momentum conservation conditions. We represent the equation of state of the medium in the form of a polynomial:

$$p^* = \alpha_1 \varepsilon^* + \alpha_2 \varepsilon^{*2}.$$

We have

$$D\frac{\partial u}{\partial \xi} + \frac{1}{\rho}\frac{\partial p}{\partial \xi} = 0, \quad D\frac{\partial v}{\partial \xi} + \frac{1}{\rho}\frac{\partial p}{\partial \eta} = 0,$$
$$D\frac{\partial p}{\partial \xi} + \rho\left(\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \eta}\right) = 0, \quad p = p^* + \beta_1(\varepsilon - \varepsilon^*) + \beta_2(\varepsilon - \varepsilon^*)^2 \tag{2}$$

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in the unloading domain in the moving coordinate system $\xi = Dt + x$, $\eta = y$, and the boundary condition has the form

$$\text{for } \eta = 0, \ \xi \geqslant 0 \quad p = f(\xi), \tag{3}$$

where $f(\xi)$ is a known monotonically decreasing function.

Let us introduce the following: D is the velocity of the moving load, a is the shock-propagation velocity, $a_1 = c_p = \beta_1/\rho$ is the propagation velocity of the unloading strain for the case $\beta_2 = 0$, p is the pressure, ε is the volume strain, Σ is the shock front, ρ is the density of the medium, t is the time, x and y are fixed Cartesian coordinates, ξ and η are moving Cartesian coordinates, V is the mass flow rate of the medium, u and v are the velocity projections on the ξ and η axes, φ is the velocity potential, v_n^* and v_τ^* are the normal and tangential components of the mass flow rate V of the medium to the front Σ, p_0 is the maximum value of the moving load, μ is a dimensionless coefficient, b is a dimensional coefficient, α_1, α_2, β_1, and β_2 are constants, α is the slope of the shock front Σ to the half-plane boundary, $\tan\alpha_0$ is the tangent of the angle of front inclination Σ to the 0ξ axis at the origin, and the parameters of the medium referring to the front Σ are denoted by a superscript asterisk.

As has been mentioned above, the method of characteristics is used to solve the problem, and the main relationships on the characteristics are presented in [3] for the case of a nonlinearly compressible medium.

The problem for a specific medium structure [4] is realized on an electronic computer for a load varying along ξ according to an exponential law of the form

$$f(\xi) = p_0 \exp(-0.1\xi)$$

where

$$\alpha_1 = 12.127\cdot 10^2, \ \alpha_2 = 58.73\cdot 10^3, \ \beta_1 = 9.016\cdot 10^3,$$
$$\beta_2 = 19\cdot 10^4, \ p_0 = 105 \text{ kg/cm}^2,$$

and the results of the computations are represented in Figs. 1-6, where the solid lines refer to the case $\alpha_2 \neq \beta_2 \neq 0$, the dashed lines to $\beta_2 = 0$, and the dashed–dot lines to $\alpha_2 = \beta_2 = 0$. The parameters of the medium in Figs. 1-3 are presented in dimensionless form relative to their maximum value and the coordinates ξ and η, relative to unit length.

It is seen from Fig. 1 that the pressure p* and the velocities u* and v* on the front damp out in an essentially nonlinear manner depending on the depth η. It turns out that each material point of the half-plane is in a more stressed state because of the nonlinearity of the medium properties than in the case $\alpha_2 = \beta_2 = 0$. The difference between the parameters computed according to linear ($\alpha_2 = \beta_2 = 0$) and nonlinear ($\alpha_2 \neq \beta_2 \neq 0$) theories is 20-30% on the average, which shows the need to take account of the nonlinear processes occurring in the medium.

Analyzing the dependences presented in Fig. 2, it can be noted that the velocity components u and v drop monotonically (the pressure is given) on the boundary of the medium $\eta = 0$ along ξ.

The curve of the dependence of the vertical velocity component v on ξ for $\alpha_2 = \beta_2 = 0$ lies above the curve referring to the case $\beta_2 = 0$, and the curve computed for the nonlinear case lies below this curve. For $\xi > 30$, when the pressure becomes negligible, the curves for v agree for $\beta_2 = 0$ and $\alpha_2 = \beta_2 = 0$. The curves for u are obtained identical to the accuracy of the line thickness everywhere on the half-plane boundary in all cases.

Curves 1-3 in Fig. 3 show the changes in the parameters of the medium at the sections $\xi = 0.5$, 1, 3, respectively, depending on the degree of wave propagation, which asymptotically reaches its maximum value (on the front) represented by the curves 1 for the two cases considered above.

Hence, let us note that the quantities p and u are obtained somewhat higher, and the curves for v have intersections, in the case of nonlinear, as compared with linear, unloading.

The change in the tangent of the angle of shock-front inclination to the medium boundary is shown in Fig. 4, where it is noted that the nonlinear properties of the material of the medium result in curving of the wave front and the front velocity damps out along the depth of the half-plane. The greatest slope for fixed ξ corresponds to the case of nonlinear loading and unloading of the medium. The curve corresponding to the case of just nonlinear loading lies below the curve of nonlinear loading and unloading, while the straight line refers to linear theory.

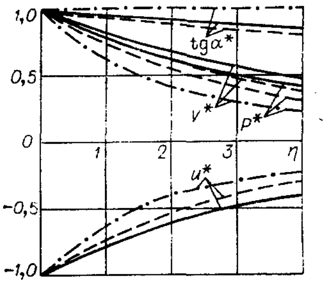

Fig. 1

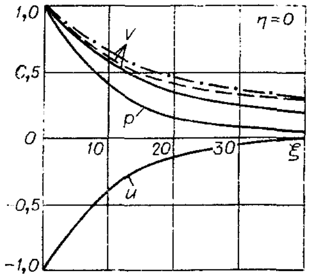

Fig. 2

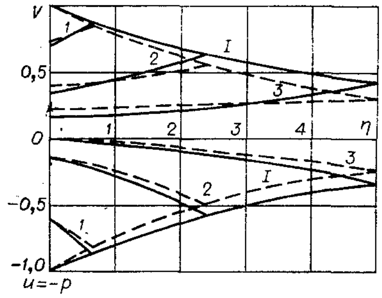

Fig. 3

Therefore, the nonlinear dependence between the parameters of the medium p and ε results in broadening of the perturbation domain.

The dependence between the maximum value of the vertical mass flow rate component v_{max} and $\tan\alpha$ of the front is presented in Fig. 5, which confirms that if the wave front tends to the boundary of the medium, then v_{max} grows.

The dashed line with circles presented in Fig. 6 corresponds to the distribution of the pressure p* along the front Σ for $\beta_2 = 0$ for the case of approximating the loading branch of the p ~ ε diagram by a chord passing through the points p = 0 and $p = p_0$. Depending on η, this pressure curve is located above the pressure curve for nonlinear loading.

Therefore, it is shown in an investigation of the influence of the nonlinear dependences between the parameters of the medium on the stress wave propagation therein by the method of characteristics that the nonlinear dependence between p and ε results in broadening of the perturbation domain – to enlargement of the pressure and velocity in comparison with linear theory. In this case, the parameters p, u, and v as well as the shock-propagation velocity in the medium under consideration become monotonically decreasing functions of the depth of the half-space.

An investigation of the system of equations (2) showed that the problem can be solved by an analytical method for $\beta_2 = 0$. Indeed, substituting the first equation of (2) into the third, we obtain a wave equation for the velocity potential φ:

$$\mu^2 \frac{\partial^2\varphi}{\partial\xi^2} - \frac{\partial^2\varphi}{\partial\eta^2} = 0 \qquad \left(\mu^2 = \frac{D^2}{c_p^2} - 1,\ c_p = \frac{\beta_1}{\rho}\right), \tag{4}$$

which has a solution of the form

$$\varphi(\xi, \eta) = f_1(\xi - \mu\eta) + f_2(\xi + \mu\eta) \tag{5}$$

for $D = c_p$. Here, the unknown functions f_1 and f_2 should be determined from the boundary condition (3) and the condition (1) on the wave front. It is proposed to solve this problem below by a reverse method, i.e., the wave front Σ is given by a definite shape of the surface and the appropriate loading profile is determined during solution of the problem. In this case we obtain a Cauchy problem for (4) within a curvilinear sector $\xi 0 \Sigma$, since if the surface of the front Σ is given (in this case the front velocity is considered a function decreasing with the depth of the half-plane), then, taking account of (1), all the parameters, including the velocity components u and v thereon of the medium, will be known variables and have the following form for $\eta = \eta(\xi)$:

$$u = \frac{\partial\varphi}{\partial\xi} = -D\sin^2\alpha(\xi)\left[\frac{\rho_0 D^2}{\alpha_2}\sin^2\alpha(\xi) - \frac{\alpha_1}{\alpha_2}\right],\quad v = \frac{\partial\varphi}{\partial\eta} = D\sin\alpha(\xi)\cos\alpha(\xi)\left[\frac{\rho_0 D^2}{\alpha_2}\sin^2\alpha(\xi) - \frac{\alpha_1}{\alpha_2}\right], \tag{6}$$

TABLE 1

ξ	u*		v*		p*	
	I	II	I	II	I	II
0	–1,644	–1,644	13,100	13,100	105	105
0,2	–1,635	–1,633	13,040	13,02	104,412	104,2
0,4	–1,622	–1,621	12,953	12,95	103,542	103,5
0,6	–1,610	–1,610	12,876	12,87	102,787	102,8
0,8	–1,598	–1,598	12,800	12,80	102,038	102,0
1,0	–1,587	–1,587	12,725	12,73	101,293	101,3

Note. I) Numerical method of characteristics; II) analytic method.

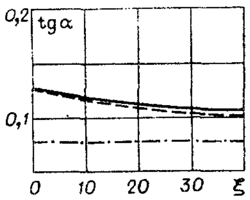

Fig. 4

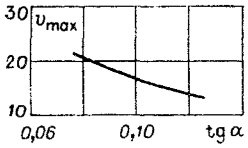

Fig. 5

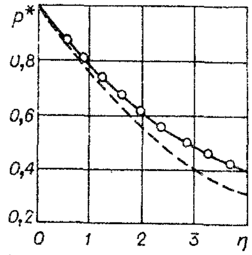

Fig. 6

TABLE 2

ξ	u		v		p	
	I	II	I	II	I	II
0	−1,644	−1,644	13,100	13,100	105	105
0,2	−1,610	−1,613	12,944	12,94	102,921	102,979
0,4	−1,581	−1,581	12,780	12,78	100,882	100,937
0,6	−1,550	−1,551	12,621	12,62	98,888	99,021
0,8	−1,519	−1,520	12,466	12,47	96,928	97,042
1,0	−1,490	−1,490	12,314	12,32	95,009	95,127

Note. I) Numerical method of characteristics, II) analytic method.

where $\eta(\xi)$ is the equation of the front surface. Using (6), from (5) we find

$$f_i(z_i) = \mp \frac{D}{2\mu} \int_0^{z_i} \frac{\operatorname{tg}\alpha\,[F_i(z_i)]\,\{1 \pm \mu \operatorname{tg}\alpha\,[F_i(z_i)]\}\,\Phi_i(z_i)}{\{1 + \operatorname{tg}^2\alpha\,[F_i(z_i)]\}^2}\,dz_i, \tag{7}$$

where $\Phi_i(z_i) = (\rho_0 D^2/\alpha_2 - \alpha_1/\alpha_2)\tan^2\alpha[F_i(z_i)] - \alpha_1/\alpha_2$; $F_i(z_i)$ is the root of the equation $\xi \pm \mu\eta(\xi) = z_i$; the upper sign in (7) is taken in the case i = 1. Therefore, taking account of (7) and (5), an analytic solution of the problem is obtained in the domain $\xi 0 \Sigma$. If this solution is substituted into (3), then a monotonically decreasing load profile with a sharp front at the origin should, in principle, be obtained and the process of unloading the medium should be realized in the perturbed domain. The results of computations show that the unloading process can be achieved in the sector $\xi 0 \Sigma$ if the velocity of the wave front Σ is a damping function over the half-plane depth (as is required to prove). An analogous reverse method can be applied in the problem of the unloading wave [5].

As an illustration of the method, the case when the surface Σ of the wave front is given in the form of a second-degree polynomial is considered:

$$\eta(\xi) = \operatorname{tg}\alpha_0 \cdot \xi - (b/2)\xi^2. \tag{8}$$

The results of computations of the analytic method with (8) taken into account for $\tan\alpha_0 = 0.1255$, $b = 0.86 \cdot 10^{-3}$ and of the method of characteristics elucidated above are presented in Tables 1 and 2, from which it is seen that the results obtained by using both methods are in satisfactory mutual agreement, and the load profile $f(\xi)$ found by the reverse method is monotonically decreasing along ξ.

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LITERATURE CITED

1. A. M. Skobeev and L. M. Flitman, "Moving load on an inelastic half-plane," Prikl. Mat. Mekh., 34, No. 1 (1970).
2. S. M. Kapustyanskii, "Two-dimensional wave propagation and reflection," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 1 (1973).
3. Kh. A. Rakhmatulin and N. Mamadaliev, "Nonlinear wave propagation in a ground half-space caused by a load traveling over its boundary," in: Symposium Transactions. Nonlinear and Thermal Effects in Transient Wave Processes [in Russian], Gor'kii–Tallin (1973).
4. "Investigation of the mechanical properties of solis under triaxial compression conditions with an elevated stress level," in: Reports of the V. V. Kuibyshev Moscow Institute of Construction Engineers [in Russian], No. 320 (1972).
5. Kh. A. Rakhmatulin and Yu. A. Dem'yanov, Strength under Intense Short-Range Loads [in Russian], Fizmatgiz, Moscow (1961).